

## What if General Lee Had a Graphing Calculator at Gettysburg?

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Every teacher these days feels the need to make the math in his courses become real and, dare we hope, even exciting to our students. It was this desire and a happy coincidence that prompted me to combine a newly-found enthusiasm for a milestone event in the history of the United States with the mathematics we study at the pre-calculus and calculus levels. One day as I stood at the green board expounding the virtues of the parabola to model real events, I found myself chatting about artillery fire. An idea then began to take shape. What if I could present a project that would allow my students to look at a key military battle and model it on the graphing calculator? Of course I could, but which battle? Well, it just so happened that the movie Gettysburg was making the rounds of selected theaters and the head of our history department was taking the entire US History class to view it. In desperate need of chaperones, he turned to me to call in a favor. This was the battle I was looking for. The movie was great and gave me a hero whose name should have been important to me much earlier. Joshua Chamberlain was a Maine man, governor of the state, president of the college I attended and a Medal of Honor winner. The movie brought out his humanism and heroism. I had a new hero and I intended to find out more.

This is probably a very good time for me to offer a serious and wide-ranging disclaimer. I am not an historian in any sense of the word. My enthusiasm for history comes late in life. It is more a product of A&E, Ken Burns, The Killer Angels by Michael Sharra and Ted Turner's movie Gettysburg. I am a very visual learner and now had at my disposal the VCR and a very knowledgeable colleague. At one time, I mentioned to this friend that I had to go stand on Little Round Top before I died. He looked at me standing there after a long day in front of the classroom and said "We better hurry and get that done." Three trips to Gettysburg and many books and videos later, I stand ready to share this presentation with you.

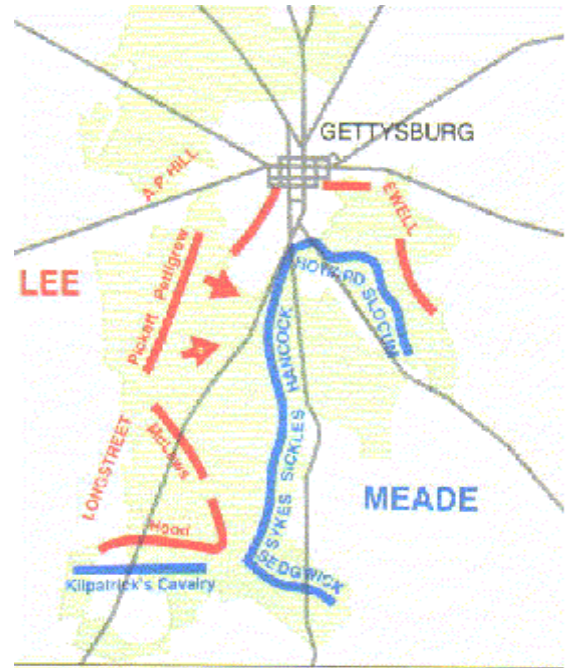
On one of these trips, we took a tour along Cemetery Ridge at what has been called the "high water" mark of the Confederacy. Our young guide was very animated as he described the artillery assault just prior to Pickett's Charge. His point, presented so enthusiastically, was that following the initial volleys, the trajectory of the cannon fire was altered and the shells began to find their way over the crest of the ridge. This point was later confirmed to me in Stars In Their Courses, by Shelby Foote. I could see it plainly now. Trajectory, vectors, parametric equations, distances, elevations, gravity, graphing calculator, physics, math and more math all joined with the spectacle of Gettysburg.

I present this workshop to students at the pre-calculus and calculus levels. A knowledge of the graphing calculator, vectors, matrices, and various other algebra and geometry skills are required.

I start my school workshop with a Hyperstudio Presentation which establishes the battlefield placement, the overall situation, the key people on both sides and the following premise: The weather conditions and soft ground at the battlefield contributed to a change in the cannon fire trajectory and a lack of observational capability that rendered the greatest artillery barrage ever on the American continent to be ineffective. This presentation will eventually be available on the Internet.

For the benefit of the readers of this paper who will not view it, I present a list of the topics in this presentation and a brief idea of the content:

- A) **Date:** July 3, 1863
- B) **Location:** Gettysburg, PA
- C) **Previous Action:** Two days previous, a group of Union cavalry had encountered a small portion of the entire Confederate Army. A battle ensued in which the aroused Confederates chased the Union soldiers through the town of Gettysburg to a defensive position on the heights on Cemetery Ridge and Culp's Hill.
- D) **Weather Report:** There had been some rain, in previous days, to soften the ground but, as the hour for Pickett's Charge approached the day was very hot and humid. The hot, humid air and the softened ground play a large role in this battle.
- E) **Placement of the troops:** The Confederates were situated on Seminary Ridge. The Union troops were on a slightly higher ridge called Cemetery Ridge. Between them lay a shallow valley of undulating ground crossed by the Emmitsburg Road and some farm fences. The distance between the ridges is almost a mile.
- F) **Key people in the battle:** Confederates: General Robert E. Lee, General James Longstreet, Col. Edwin P Alexander, General George F. Pickett, General Lewis A. Armistead. Union: General George G. Meade, General Winfield S. Hancock.
- G) **The plan of attack:** A concentrated artillery barrage is to be concentrated on the center of the Union position. Lee feels that Meade has strengthened his flanks and weakened his center position. He plans to weaken the Union in the center or even drive them from the ridge altogether. The infantry could then assault the weakened center and split the Union troops into two very much weakened forces.
- H) **Longstreet talks to Lee:** Longstreet listens to Lee's plan but feels that it will fail. He would prefer to wait for the



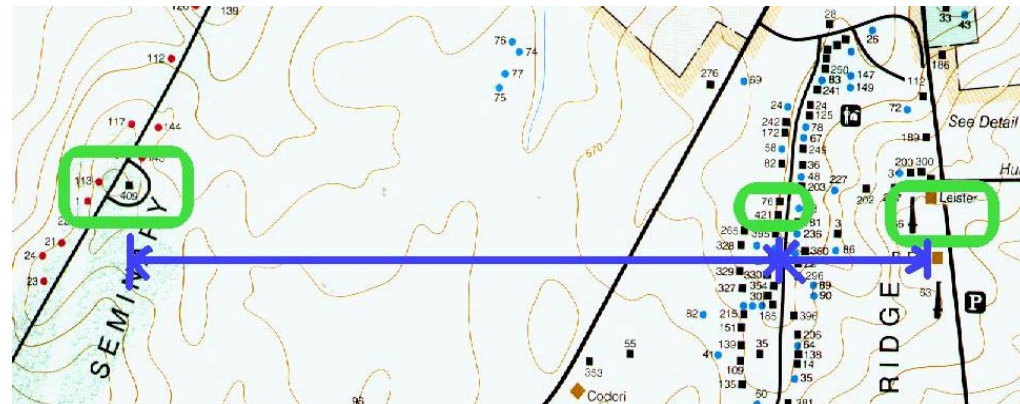
Union to attack so they could fight a defensive battle. Lee disagrees. Longstreet comes close to insubordination when he asks that someone else be placed in command of the assault.

- I) **Longstreet talks to Alexander:** Col. Alexander is the best artillery man in the Confederate Army. Longstreet directs him to place some fire on the Union batteries situated on Little Round Top. However, most of his fire, more than 150 cannon is to be directed on the ridge at the center of the Union position. He is to fire all cannon and only save enough ammunition to support the infantry on its heroic assault of the ridge almost a mile away.
- J) **Longstreet talks to Pickett:** Pickett's division has arrived on the battlefield weary from the long march but ready for a fight. Longstreet tries to be optimistic as he tells Pickett that he will lead the charge. However, he is less than confident as he asks, "George, can you take the ridge?" Pickett replies with a war whoop.
- K) **Longstreet talks to Generals:** Longstreet meets with Pickett, Trimble and Pettigrew to outline strategy. There will 15,000 men spread out over a mile. In a series of turning maneuvers, the advance will pinpoint a small clump of trees on Cemetery Ridge.
- L) **The barrage starts:** At about 1:00 in the afternoon, a two-gun signal begins what is to be the largest artillery action ever on the American continent. The Union batteries answer. Smoke hangs heavily over the battlefield, making observation difficult at best. The softened ground allows the cannon trail to sink slightly into the ground. This changes the angle of elevation of the cannon. The Union commanders decide to curtail their return fire in order to save ammunition for the infantry charge that they know will follow. They also hope that the Confederates will interpret this as a sign of withdrawal or weakness, a ruse that does have some effect. The barrage will last almost two hours.
- M) **The Confederate Infantry moves forward:** The Confederate Infantry emerges from the cover of the woods, bravely advancing through the line of cannon. My presentation concentrates on General Armistead of Virginia. He is an inspirational leader who leads his troops over the Union protective wall. He is mortally wounded by the troops commanded by his friend and companion of earlier days, General Hancock.
- N) **The Union Cannon begins its effective return fire:** As the Confederate troops come into range, a murderous fire commences from the Union cannon.

The Hyperstudio presentation is followed by video clips of the movie, illustrating many of the events mentioned here. So where is the math? Well, it is time now. The map below is used for establishing the some key distances and elevations. We use three key spots on the map:

- A) The location of the Virginia monument on Seminary Ridge. This will represent the Confederate position.
- B) The small box labeled "76" on Cemetery Ridge. This represents the Union position. It denotes the position of the 14<sup>th</sup> Connecticut Infantry.

- C) The small house on the Taneytown road behind and below Cemetery Ridge. It is the Leister House and was the location of Meade's Headquarters.



The students draw a line through these three positions and use a ruler and unit conversion techniques to establish distances in feet. By using the contour lines, we establish approximate elevations. For the purposes of this paper, we will use the values shown in these calculator windows:

The Lists

F1	F2	F3	F4	F5	F6	F7
Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	DIST	ELEV				
	c1	c2	c3	c4	c5	
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2	4231.	600.				
3	5205.	565.				
4						
5						
6						
7						
r4c2=						
MAIN		DEG APPROX		PAR		

The Window

F1	F2
Zoom	
tmin=0.	
tmax=5.	
tstep=.1	
xmin=-10.	
xmax=6000.	
xscl=100.	
ymin=560.	
ymax=700.	
yscl=10.	
MAIN	
DEG APPROX	
PAR	

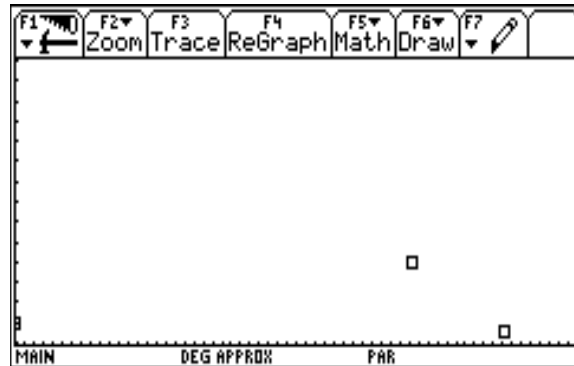
The Y= Screen

F1	F2	F3	F4	F5	F6	F7
Zoom	Edit	✓	All	Style	Plot	...
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Plot 2:						
Plot 1: [x] c1 [y] c2						
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MAIN		DEG APPROX		PAR		

The Plot Setup

F1	F2
main\3etty Plot 1	
Plot Type.....	Scatter→
Mark.....	Box→
x.....	c1
y.....	c2
Plot. Symbol Width:	1
Use Freq and Categories?	NO→
Freq.....	
Category.....	
(Include Categories?)	C
Enter=SAVE	
ESC=CANCEL	
USE ← AND → TO OPEN CHOICES	

### The Resulting Plot



In our previous course work we will have established all the foundations for the math we will do next. Of prime importance is work in vectors, matrices and parametric equations.

To establish the plot of cannon fire a pair of parametric equations needs to be established. In their general form the look like this:

$$X_{IT} = V_0 T * \cos(\alpha)$$

$$Y_{IT} = -g/2 T^2 + V_0 T * \sin(\alpha) + H_0$$

where:

- $V_0$  is the Initial Velocity or Muzzle Velocity
- $g$  is the coefficient of acceleration due to gravity
- $H_0$  is the Initial Height (Elevation)
- $\alpha$  is the Angle of Elevation for the Cannon

In my research, I could not find a muzzle velocity for any of the cannon. I did find, however, enough data to solve for a good approximation for this muzzle velocity. A 12-lb cannon ball has a range of 2100 yards at 4 degrees of elevation and a time-in-the-air of 5 seconds. Using the equation for  $X_{IT}$ , we can solve for  $V_0$ .

$$X_{IT} = V_0 T * \cos(\alpha)$$

$$2100(3) = V_0 ( 5 ) \cos( 4 )$$

$$V_0 = 6300/(5 \cos(4)) \nabla 1263 \text{ ft/sec}$$

If we accept the gravity factor to be  $-32 \text{ ft/sec}^2$ , we can now establish our trajectory equations:

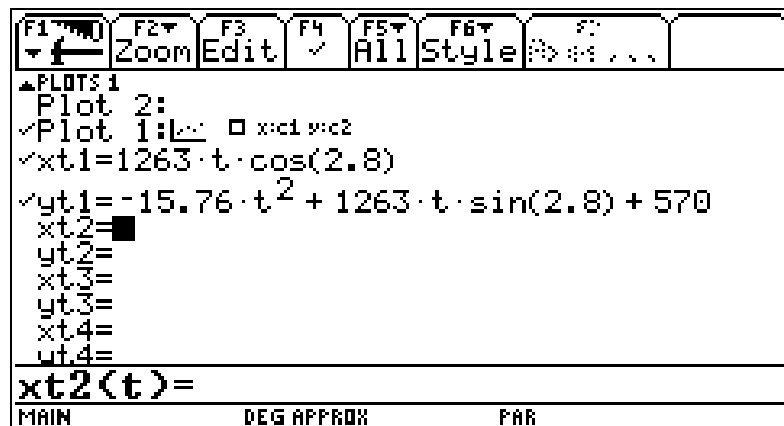
$$X_{IT} = 1263 T * \cos(\alpha)$$

$$Y_{IT} = -32/2 T^2 + 1263 T * \sin(\alpha) + 570$$

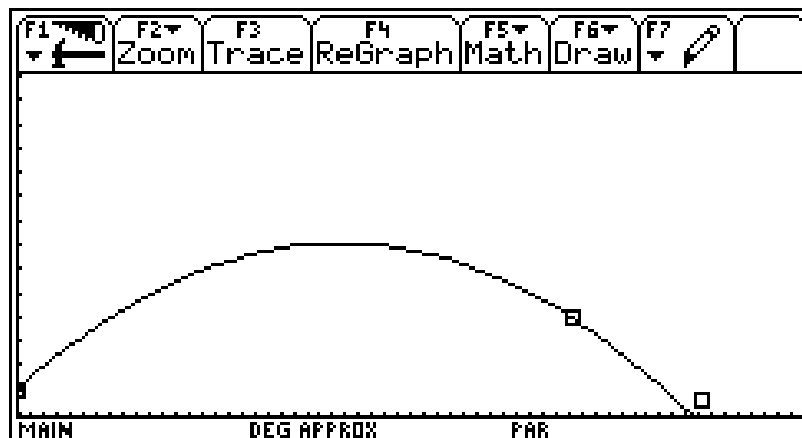
At one point a leader of a workshop I was attending gave me an idea of how to expand this with a project using the Calculator Based Lab or Ranger. He very pointedly asked me if the battle was fought in a vacuum. This prompted many experiments dropping various sized balls on the CBR to allow us to compute the gravity factors related to size and weight. This culminated in a ritual dropping of a cannon ball onto the CBR, in a protected case of course. In addition to that, we borrowed a projectile launcher from our friends in the science department. They have all the nicest academic toys. By the time we are ready to do this workshop, all of this math and calculator work is second nature.

After our experimentation with the CBR we settled on a value for  $-g/2$  as  $-15.76$ . This value will change slightly with each class. Students are now encouraged to find an angle of elevation which will land a cannon ball squarely on the Union position. This often is an exercise in what an appropriate estimation is. The occasional estimation of  $80^\circ$  is quickly discouraged even if it lands close.

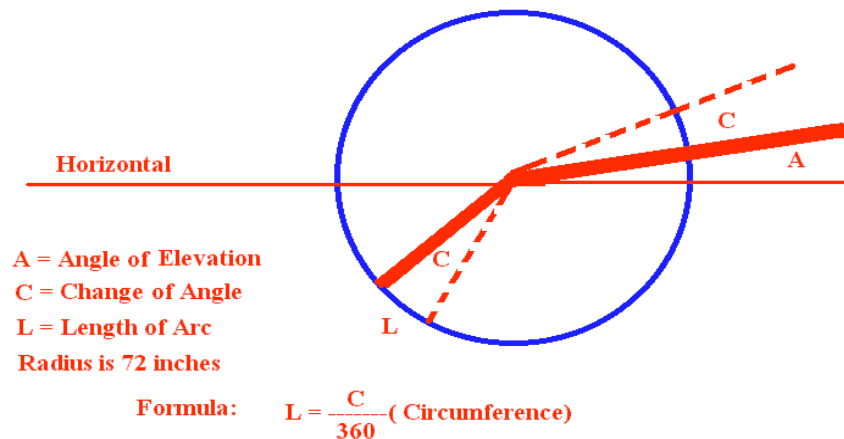
The Y= Screen



The First Trajectory Plot



Now comes the unexpected factor that may have changed the course of history: The trail, rear stabilizing portion of the cannon, sinks almost imperceptibly into the soft soil. The cannon balls disappear into the smoke created by extensive cannon fire from opposing ridges of the battlefield. Taking a good deal of academic license, I suggest that this change is the tiny value of 0.175 inches. Time to translate this to the angular change in the elevation of the cannon. The cannon trail is 72 inches. Therefore, the circumference of this circle used here as a model is  $144\pi$  inches.



Computation of Change of Elevation

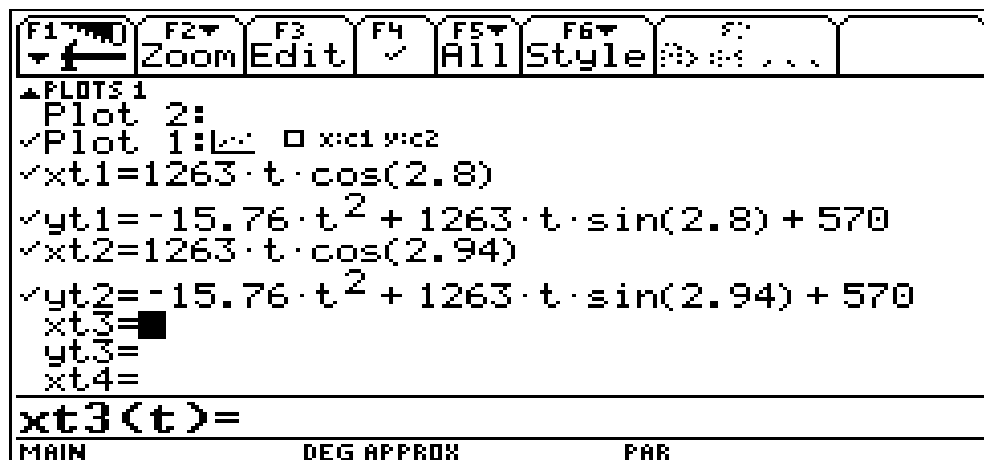
$$0.175 = (C/360) (144\pi)$$

$$C = (0.175 * 360) / 144\pi$$

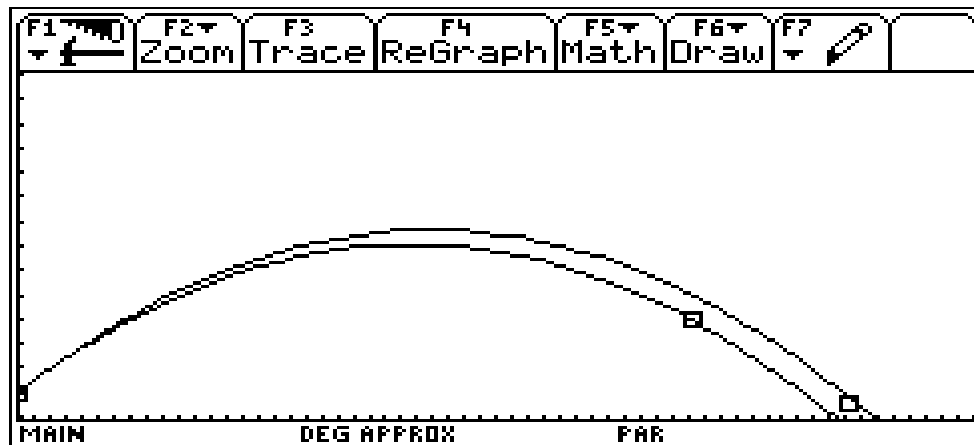
$$C \nabla 0.14^\circ$$

Students will be very eager to enter this new angle of elevation into their calculators. The results become obvious. This slight increase in elevation has caused the cannon fire to overshoot the Union troops and fall ineffectively on the backside of the slope. Indeed, the fire was so devastating at the Leister House that General Meade was forced to move his headquarters.

The New Y= Screen



### The New Trajectory Screen



This proves to be very unfortunate for the Confederate troops. As they enter onto the open battlefield, they soon realize that their enemy has not been driven from the ridge nor has it been weakened significantly. Pickett's Charge, while heroic in effort and proportion, is destined to be a terrible defeat from which the Confederacy never recovers. All that remains for us is some video of the advance of Pickett's men. The presentation concludes when the Union guns begin their murderous fire.

### References

The Killer Angles by Michael Sharra

Stars In Their Courses by Shelby Foote

Gettysburg, Ted Turner Pictures